Indian Statistical Institute
B.Math. (Hons.) III Year
First Semester 2006-07
Mid-Sem Examination
Introduction to Differential Geometry

Time: 3 hrs Date:20-09-06 Max. Marks: 40

Instructor: Maneesh Thakur

Attempt all the problems.

1. Let γ be a plane curve with parameter s. Show that

$$\dot{\mathbf{n}}_s = -k_s \mathbf{t}$$

where n_s is the signed unit normal and $\mathbf{t}$ is the tangent vector of γ, k_s is the signed curvature of γ.

2. (A) Let γ be a unit speed plane curve such that its tangent vector $\mathbf{t}(s)$ makes a constant angle θ with $\gamma(s)$ for all s. Show that
   (i) if $\theta = 0$ then γ is part of a straight line.
   (ii) if $\theta = \pi/2$ then γ is part of a circle

   OR

   (B) Show that the reflection of a plane curve, with respect to a straight line, has signed curvature with opposite sign as that of the given curve.

3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a smooth map. Show that $\Gamma = \text{graph}(f)$ is a smooth surface contained in $\mathbb{R}^3$.

4. Let $\sigma(u, v)$ be a surface patch with first fundamental form $\mathcal{F}_1 = \begin{pmatrix} E & F \\ F & G \end{pmatrix}$ and second fundamental form $\mathcal{F}_2 = \begin{pmatrix} L & M \\ M & N \end{pmatrix}$. Let K denote the Gaussian curvature of σ and H the mean curvature. Show that
   (i) $K = \frac{LN - M^2}{EG - F^2}$
   (ii) $H = \frac{LG - 2MF + NE}{2(EG - F^2)}$

OR

5. Calculate the first fundamental form of the sphere with radius R and hence find its area.
(use the surface patch $\sigma(\theta, \phi) = (\cos\theta\cos\phi, \cos\theta\sin\phi, \sin\theta)$).